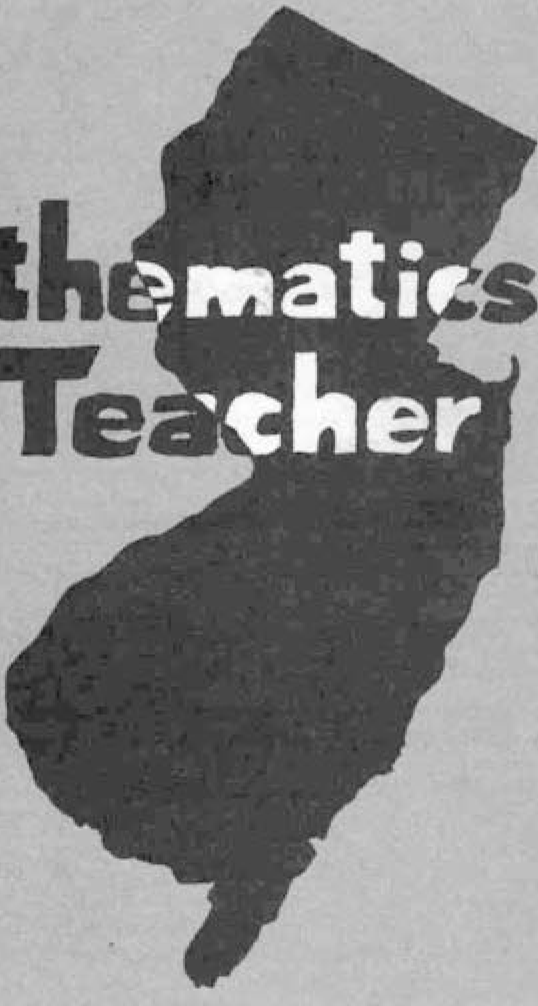
The New Jersey

Mathematics Teacher



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ASSOCIATION OF MATHEMATICS TEACHERS OF NEW JERSEY

The object of this Association shall be to encourage an active interest in mathematical science, to afford a medium of interchange of views regarding the teaching of mathematics, and to further the cooperative study of problems relating to the teaching of mathematics.

Membership in this Association shall be open to persons who are engaged in teaching of mathematics in educational institutions, public or private, or who are interested in the teaching of mathematics.

The annual dues of the Association shall be: for all active members, five dollars; for junior members (students) two dollars.

Manuscripts for publication in The New Jersey Mathematics Teacher should be typewritten, double-spaced with wide margins on 8½" x 11" paper. Two copies are required. All diagrams and tables should be done in black or red ink and be ready for photo reproduction.

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AYYADURAI'S FOUR POINT THEOREM

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(In the course of an independent study project Shiva Ayyadurai, a Junior at Livingston High School, became interested in Ptolemy's Theorem. He was at the time studying Vector Geometry and made an attempt to prove this theorem by vector methods. Shiva did not accomplish this yet but he did run across the following interesting result. -- Ed.)

In the course of investigating Ptolemy's Theorem, I encountered a most interesting relationship. Ptolemy's Theorem states that in a cyclic quadrilateral, the sum of the products of the magnitudes of the opposite sides is equal to the product of the magnitudes of the diagonals. By using vector methods, I discovered that the Dot products of the diagonals is equal to the sum of the Dot products of the opposite sides; furthermore, this relationship is true for all quadrilaterals, and, more generally, it is applicable to any four points in space. A Dot product is a scalar quantity, obtained when two vectors are multiplied.

Proof:

In any quadrilateral, the sum of the Dot products of the opposite sides is equal to the Dot products of the diagonals. Refer to Fig. 1.

$$\vec{AC} = \vec{AP} + \vec{PC}$$
$$\vec{BD} = \vec{BP} + \vec{PD}$$
$$\vec{AC}\cdot\vec{BD} = (\vec{AP} + \vec{PC})\cdot(\vec{BP} + \vec{PD})$$

$$\vec{AB} = \vec{AP} - \vec{BP}$$
$$\vec{CD} = \vec{PD} - \vec{PC}$$
$$\vec{AB}\cdot\vec{CD} = (\vec{AP} - \vec{BP})\cdot(\vec{PD} - \vec{PC})$$
$$= \vec{AP}\cdot\vec{PD} - \vec{AP}\cdot\vec{PC} - \vec{BP}\cdot\vec{PD} + \vec{BP}\cdot\vec{PC}$$

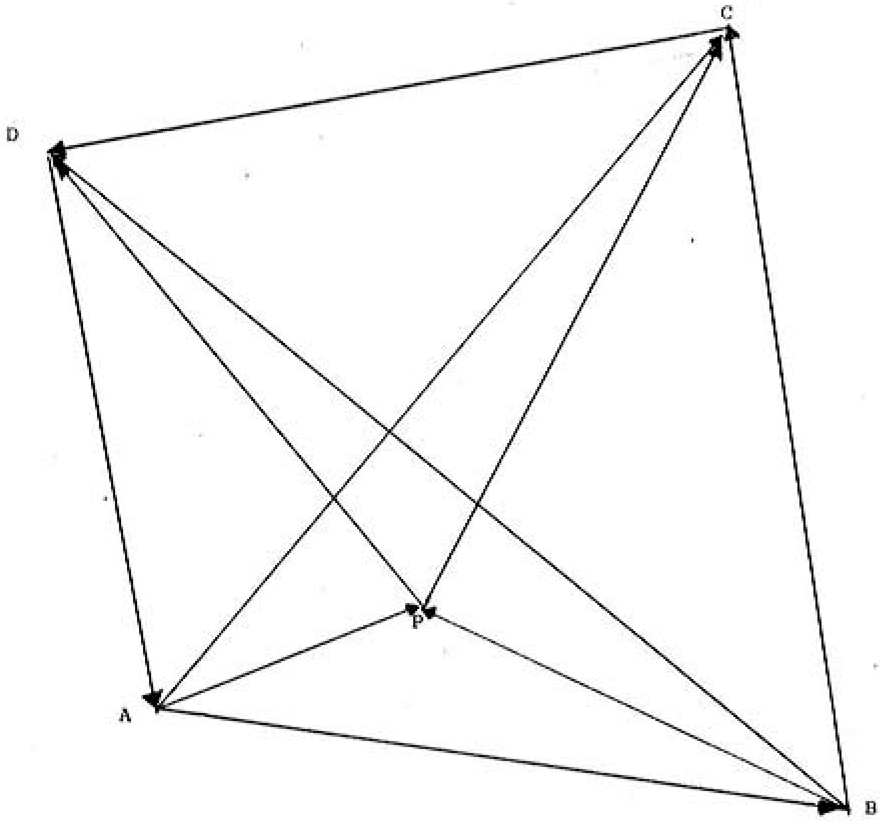
$$\vec{AD} = \vec{AP} + \vec{PD}$$
$$\vec{BC} = \vec{BP} + \vec{PC}$$
$$\vec{AD}\cdot\vec{BC} = (\vec{AP} + \vec{PD})\cdot(\vec{BP} + \vec{PC})$$
$$= \vec{AP}\cdot\vec{BP} + \vec{AP}\cdot\vec{PC} + \vec{PD}\cdot\vec{BP} + \vec{PD}\cdot\vec{PC}$$

Add the Dot products of $\vec{AB}\cdot\vec{CD}$ and $\vec{AD}\cdot\vec{BC}$:

$$\vec{AB}\cdot\vec{CD} + \vec{AD}\cdot\vec{BC} = \vec{AP}\cdot\vec{PD} + \vec{AP}\cdot\vec{BP} + \vec{BP}\cdot\vec{PC} + \vec{PD}\cdot\vec{PC}$$
$$= \vec{AP}\cdot(\vec{PD} + \vec{BP}) + \vec{PC}\cdot(\vec{BP} + \vec{PD})$$
$$= (\vec{AP} + \vec{PC})\cdot(\vec{PD} + \vec{BP})$$
$$= \vec{AC}\cdot\vec{BD}$$
$$\therefore \vec{AB}\cdot\vec{CD} + \vec{AD}\cdot\vec{BC} = \vec{AC}\cdot\vec{BD}$$

As I have stated before, this theorem is valid for any four points in space. They can be collinear, coplanar, or non-coplanar. In fact, the points need not necessarily be distinct.

Figure 1



Note: Point P may be selected anywhere in space, to prove the theorem. Since no restriction is placed on the other four points: A, B, C, D. This theorem is applicable to any four points in space.

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